Feuille d'exercices 2 : Systèmes linéaires

Exercice 1:

Résoudre les systèmes suivants dans $\mathbb{R}^3$:

$$a)\left\{\begin{array}{lcl} 2x-y & = & 0 \\ -x+y+z & = & 0 \\ -x+3y-2z & = & 0 \end{array}\right. \quad b)\left\{\begin{array}{ccc} x-y+z & = & 1 \\ 2x+y-z & = & 2 \\ x-2y+3z & = & 0 \end{array}\right. \quad c)\left\{\begin{array}{ccc} 2x+3y+z & = & 4 \\ -x+y+2z & = & 3 \\ 7x+3y-5z & = & 2 \end{array}\right. \quad d)\left\{\begin{array}{lcl} x+2y-z & = & 0 \\ 2x \quad -z & = & 0 \\ x-2y & = & 0 \end{array}\right.$$

$$e)\left\{\begin{array}{ccc} 2x-y+z & = & 3 \\ x+2z & = & -1 \\ x-y-z & = & 2 \end{array}\right. \quad f)\left\{\begin{array}{ll} 4x+2y-2z & =0 \\ 3x-y+z & =3 \\ x+y+z & =1 \\ x-y+z & =-2 \end{array}\right. \quad g)\left\{\begin{array}{ll} 3x-6y-6z & =0 \\ x-2y-3z & =0 \\ -2x+4y+6z & =0 \\ 6x-12y-12z & =0 \end{array}\right.$$

Exercice 2:

Résoudre les systèmes suivants dans $\mathbb{R}^4$:

$$a)\left\{\begin{array}{ccc} -3x+y+z+t & = & 0 \\ x-3y+z+t & = & 0 \\ x+y-3z+t & = & 0 \\ x+y+z-3t & = & 0 \end{array}\right. \quad b)\left\{\begin{array}{ccc} -x+3y-t & = & 0 \\ 2x-y+2z+2t & = & 0 \\ 5y+2z & = & 0 \\ x+2y+2z+t & = & 0 \end{array}\right. \quad c)\left\{\begin{array}{ll} x+y+z+t & =1 \\ x+y-z-t & =-1 \end{array}\right.$$

Pour ceux qui veulent augmenter la difficulté des calculs :

$$d)\left\{\begin{array}{lcl} \quad -y+2z+3t & = & 0 \\ 2x+2y-z & = & 0 \\ 3x-y+2z-2t & = & 0 \\ 5x+y+z-2t & = & 0 \end{array}\right.$$

Exercice 3:

A quelles conditions portant sur les paramètres a, b, c et d, les systèmes suivants sont-ils compatibles ? Le cas échéant, finir la résolution des systèmes.

$$(S_1)\left\{\begin{array}{ccc} x+y+z & = & a \\ x-y-z & = & b \\ -3x+y+3z & = & c \end{array}\right. \quad (S_2)\left\{\begin{array}{ccc} 3x-3y-2z & = & a \\ -4x+4y+3z & = & b \\ 2x-2y-z & = & c \end{array}\right. \quad (S_3)\left\{\begin{array}{ccc} -2x-3y+3z & = & a \\ x+2y-z & = & b \\ x+y-2z & = & c \end{array}\right.$$

Pour ceux qui veulent augmenter la difficulté des calculs :

$$(S_4)\left\{\begin{array}{ccc} 2x+y-3z & = & a \\ 3x+y-5z & = & b \\ 4x+2y-z & = & c \\ x-7z & = & d \end{array}\right.$$